Synchronization of chaotic erbium-doped fiber dual-ring lasers by using the method of another chaotic system to drive them

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A method of chaotic synchronization is presented in this paper that uses the chaotic output of one system to drive two other identical chaotic systems. The criterion is defined that, when the maximum conditional Lyapunov exponents (MCLE's) of the two systems are negative, the two systems can be synchronized to each other. As a possible application we numerically investigated the synchronization of chaotic erbium-doped fiber dual-ring laser systems. Numerical calculation shows that when driven by another chaotic system, if the two identical systems are in chaos and their MCLE's, are negative, they can go into chaotic synchronization whether or not they were in chaotic states previously. Simultaneously, we find that the states of the two systems vary with that of the driving system. When the driving system is in different periodic states, the two systems can still retain synchronization and go into corresponding different periodic states.

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I. INTRODUCTION

The investigation of chaotic synchronization has attracted much attention in the last ten years due to the possibility of practical applications of this fundamental phenomenon. The concept of synchronized chaos was first introduced by Pecora and Carroll [1] (so it is called the PC method). They showed theoretically that the chaotic dynamics of two systems can be synchronized. Using an electric circuit, they also demonstrated experimentally the chaos synchronization of their concept [2]. In their scheme the synchronization is achieved in the following way. A nonlinear chaotic system is separated into two subsystems: one of which is stable, the other is not. Then the stable subsystem is replicated to produce a nonautonomous second subsystem driven by the corresponding signals in the parent (master) system. This method shows that the two subsystems become asymptotically synchronized if all the conditional Lyapunov exponents of the replicated subsystem are negative. For many chaotic dynamical systems, however, it is not easy in practice to separate the system.

In this paper we propose a method of chaotic synchronization based on the PC method. In this method we use three systems instead of the master and two subsystems in the PC method, among which one is in a chaotic state (called the driving system), and the other two are identical (called the driven systems). Then we use the chaotic output of the driving system to drive the two driven systems with the same driving stiffness. By adjusting the driving stiffness properly, when the two driven systems are in chaos and their maximum conditional Lyapunov exponents MCLE's are negative, the two driven systems can synchronize asymptotically whether or not they were in chaos previously.

Here we apply this method to erbium-doped fiber dual-ring lasers and investigate their chaotic synchronization. Erbium-doped fiber lasers have recently received a great deal of attention because of their importance in optical communications: Their wavelength (about 1.54 μs) is in the third window of optical communications. As well as applications in devices, the dynamics of erbium-doped fiber lasers with various kinds of cavity have been investigated [3–11]. More recently, there have been reports on their chaotic synchronization [12–15] using the method of the master-slave or mutual coupling synchronization. Here we use this method to investigate the conditions for chaotic or periodic synchronization in erbium-doped fiber dual-ring lasers.

II. A SCHEME OF SYNCHRONIZATION

An erbium-doped fiber dual-ring laser is made up of two coupled fiber ring lasers. The scheme is shown in Fig. 1. As we know [11], the two erbium-doped fiber lasers are each in a steady state. When they are coupled to each other through the directional coupler, they can go into periodic states. With changes of some parameter, the system can develop chaos following a period-doubling route. To obtain chaos synchronization of such systems, we use the proposed method and put forward the scheme shown in Fig. 2.

In the scheme, $S1$ is the driving system and $S2$,$S3$ are two identical driven systems. Through a directional coupler, some of the output of $S1$ in ring a (Fig. 1) is coupled to the

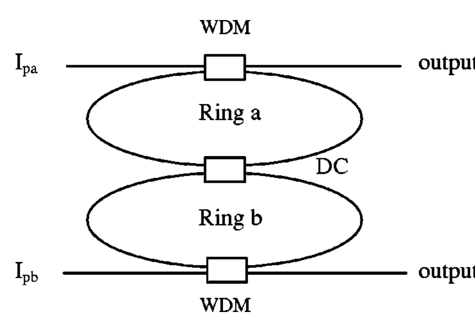

FIG. 1. Erbium-doped fiber dual-ring laser system. I_{pa},I_{pb}, pump light; DC, directional coupler; WDM, wavelength division multiplexer.

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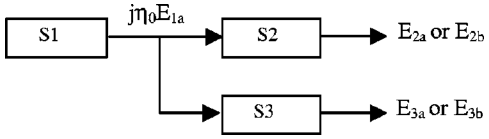

FIG. 2. Synchronization scheme: E_{2a} will synchronize to E_{3a}. $S1$,$S2$,$S3$ are chaotic erbium-doped fiber dual-ring lasers.

two driven systems in ring a and realizes the driving. Because $S1$ is not affected by $S2$ and $S3$, its dynamic equations keep their original form and are as follows [11]:

$$\dot{E}_{1a}=-\kappa_{1a}(E_{1a}+\eta_0E_{1b})+g_{1a}E_{1a}D_{1a}, \tag{1}$$

$$\dot{E}_{1b}=-\kappa_{1b}(E_{1b}-\eta_0E_{1a})+g_{1b}E_{1b}D_{1b}, \tag{2}$$

$$\dot{D}_{1a}=-(1+I_{1pa}+|E_{1a}|^2)D_{1a}+I_{1pa}-1, \tag{3}$$

$$\dot{D}_{1b}=-(1+I_{1pb}+|E_{1b}|^2)D_{1b}+I_{1pb}-1. \tag{4}$$

Here, E_{1a} and E_{1b} are the lasing fields of $S1$ in rings a and b, respectively; $|E_{1a}|^2$ and $|E_{1b}|^2$ represent the lasing intensities of $S1$ in rings a and b, respectively; D_{1a} and D_{1b} are the population inversions of $S1$ in rings a and b, respectively; κ_{1a} and κ_{1b} are the products of τ_2 and the decay rate of $S1$ in rings a and b, respectively; g_{1a} and g_{1b} are, respectively, the products of τ_2 and the gain coefficient of $S1$ in rings a and b; τ_2 is the lifetime of the population of the lasing upper level in erbium. I_{1pa} and I_{1pb} are the pump intensities in the respective fiber ring lasers and η_0 is the coupling coefficient of the directional coupler DC at the wavelength $\lambda=1.54$ μm. In the dynamic equations the time is normalized to τ_2.

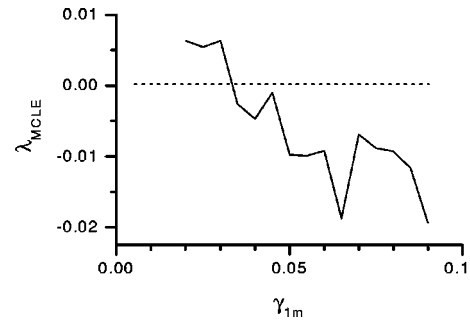

FIG. 3. The MCLE of the driven systems changes with the driving stiffness, $\lambda_{MCLE}8$ vs γ_{1m}.

For the driven systems $S2$ and $S3$, the dynamics are influenced by the driving system $S1$. In this scheme the output of $S1$ is coupled to $S2$ and $S3$ through a directional coupler and a wavelength division multiplexer (WDM). The WDM is used to input the pump light and output the laser and the

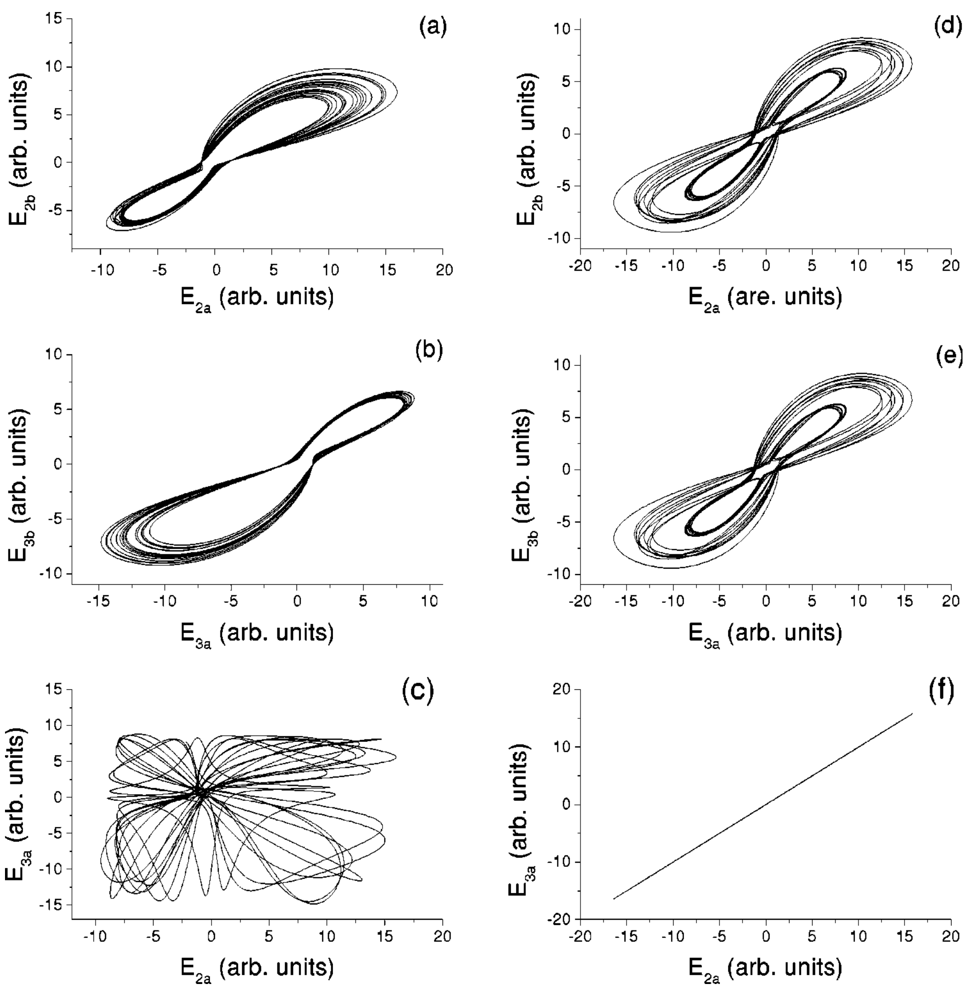

FIG. 4. Synchronized chaos for $\gamma_{1m}=0.08$. (a),(b),(d),(e) The strange attractor in the plane E_{2a},E_{2b} and E_{3a},E_{3b} before and after driving. (c),(f) The projection of the flow onto the E_{2a}-E_{3a} plane before and after driving.

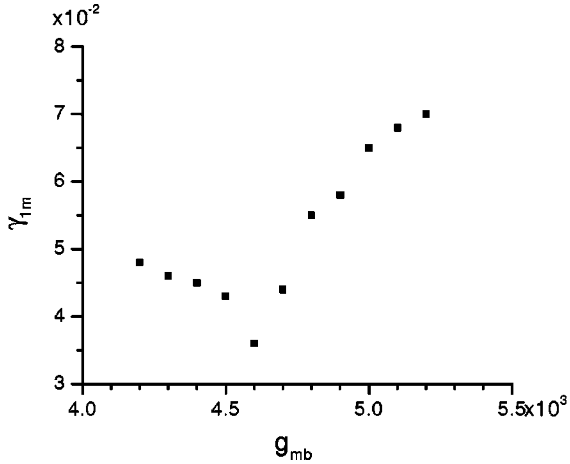

FIG. 5. The minimum driving stiffness changes with the states of the driven systems, γ_{1m}vs g_{mb}.

directional coupler is used to couple the output of $S1$ into $S2$ and $S3$. When the optical field in one fiber is coupled to the others through the WDM or directional coupler, there is a phase changes of $\pi/2$. So the fields coupled to $S2$ and $S3$ have a phase changes of π. Under this condition, the dynamic equations of $S2$ and $S3$ are changed as follows:

$$\dot{E}_{ma}=-\kappa_{ma}(E_{ma}+\eta_0 E_{mb}+\gamma_{1m}E_{1a})+g_{ma}E_{ma}D_{ma}, \tag{5}$$

$$\dot{E}_{mb}=-\kappa_{mb}(E_{mb}-\eta_0 E_{ma})+g_{mb}E_{mb}D_{mb}, \tag{6}$$

$$\dot{D}_{ma}=-(1+I_{mpa}+|E_{ma}|^2)D_{ma}+I_{mpa}-1, \tag{7}$$

$$\dot{D}_{mb}=-(1+I_{mpb}+|E_{mb}|^2)D_{mb}+I_{mpb}-1. \tag{8}$$

Here $m=2,3$, represent the two driven systems $S2,S3$, respectively; $\gamma_{1m}=\eta_1\times\eta_2$ is the driving stiffness, where η_1 and η_2 are the coupling coefficents of the WDM and the coupler at the wavelength $\lambda=1.54$ μm; E_{ma} and E_{mb} are the lasing fields of $S2$ or $S3$ in rings *a* and *b*, respectively; $|E_{ma}|^2$ and $|E_{mb}|^2$ represent the lasing intensities of $S2$ and $S3$, respectively; D_{ma} and D_{mb} are the population inversions of $S2$ or $S3$ in rings *a* and *b*, respectively; κ_{ma} and κ_{mb} are the products of τ_2 and the decay rate of $S2$ or $S3$ in rings *a* and *b*, respectively; g_{ma} and g_{mb} are, respectively, the products of τ_2 and the gain coefficient of $S2$ or $S3$ in rings *a* and *b*; I_{mpa} and I_{mpb} are the pump intensities in the corresponding fiber ring laser. The meanings of the other symbols are the same as above. Again, in the dynamic equations the time is normalized to the lifetime of the lasing upper level τ_2.

III. NUMERICAL SIMULATION

To eclucidate the process of chaos synchronization, we numerically analyze erbium-doped fiber dual-ring lasers in two cases. One is that the driving system remains in chaos and the driven systems are in various states. The other is that the driven systems remain in chaos and the driving system is in various states.

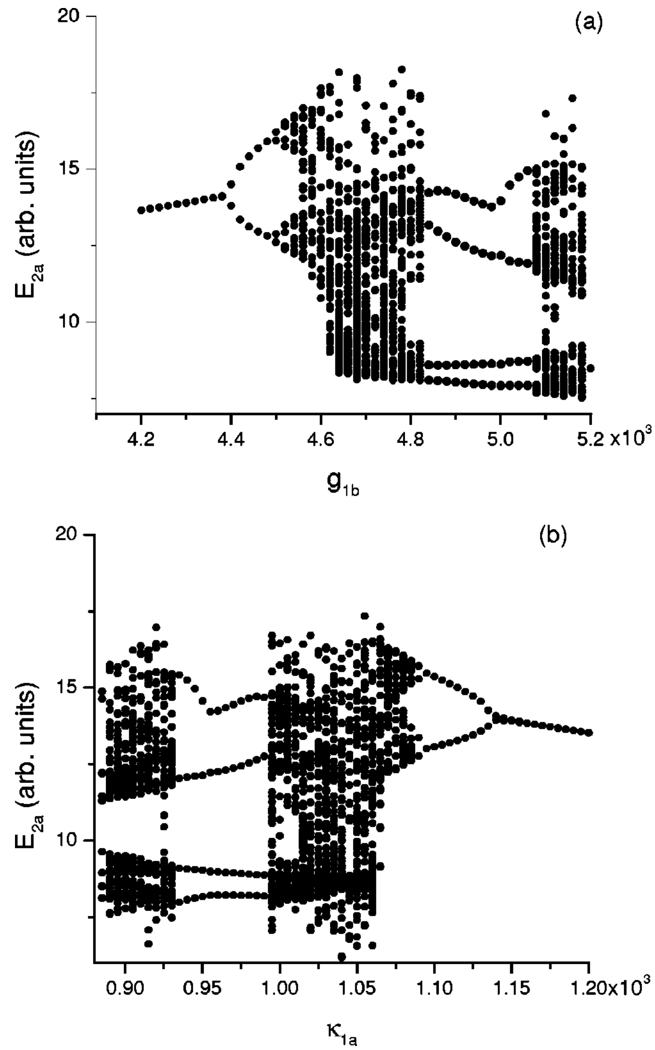

FIG. 6. The bifurcation diagram of the synchronized systems. (a) E_{2a}vs g_{1b}; (b) E_{2a}vs κ_{1a} ($\kappa_{1a}=\kappa_{1b}$).

First, we keep the driving system $S1$ in chaos and adjust the states of the two driven systems $S2$ and $S3$. As we know, the state of the erbium-doped fiber dual-ring laser can be changed by adjusting the gain coefficient of ring *b* [11] or the decay rates of ring *a* and ring *b* [15]. Here we change the states of the two driven systems by adjusting g_{mb}. In the numerical simulation, $S1$ is in chaos and we take the parameters as [11] $\kappa_{1a},\kappa_{1b}=1000$; $g_{1a}=10\,500$; $g_{1b}=4800$; $\eta_0=0.2$; $I_{1pa},I_{1pb}=4$; $\tau_2=10$ ms. For $S2$ and $S3$, in order to make them in various states, we let g_{mb} vary from 4200 to 5200 and the other parameters are [11] $\kappa_{ma},\kappa_{mb}=1000$; $g_{ma}=10\,500$; $\eta_0=0.2$; $I_{mpa},I_{mpb}=4$. Now we simulate the whole system in the scheme by using the fourth order Runge-Kutta-Gill method in double precision. The numerical results show that, whether the systems $S2,S3$ are in periodic or chaotic states previously, when they are driven by $S1$ with γ_{1m} more than 0.001, they can go into chaos. Under these conditions, as long as their MCLE's are negative, the two driven systems $S2$ and $S3$ can go into chaotic synchronization, that is, $E_{2a}=E_{3a}$, $E_{2b}=E_{3b}$. Now we take $g_{mb}=4600$. With this, the MCLE's of the two identical systems

$S2$ and $S3$ are 1.36×10^{-2} if they are not driven. This shows that $S2$ and $S3$ were in chaos previously. Then we calculate the MCLE's of the systems $S2$ and $S3$ with the driving system to drive them. Figure 3 gives the relation between the MCLE ($\lambda_{\mathrm{MCLE's}}$) and γ_{1m}. From it, we find that when the driving stiffness γ_{1m} is more than 0.035, the MCLE's are negative. This means that the two driven systems can synchronize with each other. The synchronization phenomenon can be clearly seen in Fig. 4 in which $\gamma_{1m}=0.08$. Figures 4(a), 4(b), 4(c) and 4(d), 4(e), 4(f) show the attractors of $S2$ and $S3$ and their relations before and after they are driven. These figures show that the attractors of the driven systems are influenced by the driving system. In addition, if the states of the driven systems vary, the minimum of γ_{1m} must vary with them in order to retain the chaotic synchronization. These relations are shown in Fig. 5.

Secondly, we investigate the synchronization under the conditions that $S2$,$S3$ are in chaos and $S1$ is in various states. Here we take the parameters of $S2$ and $S3$ as those of $S1$ in the first case, and the parameters of $S1$ are those of $S2$ and $S3$ in the first case. The numerical results show that the states of $S2$ and $S3$ vary with that of $S1$, and the bigger γ_{1m} is, the more they are affected. When γ_{1m} is more than 0.088, their states are the same as that of $S1$, including periodic, chaotic, and developed chaotic motion. Moreover, they can still remain synchronized. These results show that we can obtain synchronized systems with various states by changing the state of the driving system. Figures 6(a) and 6(b) respectively show the changes in the synchronized states via that of $S1$ caused by adjusting g_{1b} and κ_{1a},κ_{1b} ($\kappa_{1a}=\kappa_{1b}$) with $\gamma_{1m}=0.09$.

Based on the results above, we extended the chaotic synchronization of two systems to more systems. Here, we present one example. In this case, there are three driven systems as well as the driving system and their parameters are the same as above but with different initial conditions. The numerical results show that the three driven systems can still synchronize if their MCLE's are negative. This is significant in realizing secure communications via chaos synchronization.

IV. CONCLUSION

In conclusion, we have demonstrated a method of chaotic synchronization by chaotic driving. Using the MCLE as criterion, we give the conditions for realizing chaotic synchronization. This method has the following advantages. (1) It can make several systems synchronized. (2) By changing the state of the driving system, we can obtain periodic or chaotic synchronized systems. (3) Compared with the PC method, it is more practical since it does not require the system to be divided. Using this method, we present a scheme of chaotic synchronization in erbium-doped fiber dual-ring lasers. Numerical results verified these conclusions. At the same time, we give the bifurcation diagram of the systems synchronized via the driving system. These results have significance in practice.

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